Classification Trees: C4.5

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1 Tree Generation: Gain criteria.

Imagine selecting one example at random from a set T of training examples and announcing it belongs to the class $C_i,\ i = 1, \ldots, k$. This message has the probability:

$$\frac{|C_i|}{|T|} \tag{1}$$

. Where $|C_i|$ is the number of examples which belongs to the class C_i. The information contained in this announce is:

$$-\log_2 \frac{|C_i|}{|T|} \text{bits.} \tag{2}$$

In general, if we are given a probability distribution $P = (p_1 = \frac{|C_1|}{|T|}, p_2 = \frac{|C_2|}{|T|}, .., p_k = \frac{|C_k|}{|T|})$ then the Information conveyed by this distribution (also called the Entropy of P), is:

$$I(P) = -\sum_{i=1}^{k} p_i * \log(p_i) \tag{3}$$

When applied to the set T of training example, $I(P)$ measures the average amount of information (bits) needed to identify the class of an example of T (it's a weighted sum of the k different equations 2).

For example, if P is $(0.5, 0.5)$ then $I(P)$ is 1, if P is $(0.67, 0.33)$ then $I(P)$ is 0.92, if P is $(1, 0)$ then $I(P)$ is 0. [Note that the more uniform is the probability distribution, the greater is its information.]

An other example: We want to known if a golfer will play or not based on the following data:

```
OUTLOOK | TEMPERATURE | HUMIDITY | WINDY | PLAY
=================================================
sunny   |      85     |    85    | false | Don't Play
sunny   |      80     |    90    | true  | Don't Play
overcast|      83     |    78    | false | Play
rain    |      70     |    96    | false | Play
rain    |      68     |    80    | false | Play
```

```
rain    |      65     |    70    | true  | Don't Play
overcast|      64     |    65    | true  | Play
sunny   |      72     |    95    | false | Don't Play
sunny   |      69     |    70    | false | Play
rain    |      75     |    80    | false | Play
sunny   |      75     |    70    | true  | Play
overcast|      72     |    90    | true  | Play
overcast|      81     |    75    | false | Play
rain    |      71     |    80    | true  | Don't Play
```

We will build a classifier which, based on the features OUTLOOK, TEMPERATURE, HUMIDITY and WINDY will predict wether or not the golfer will play. There is 2 classes: (play) and (don't play). There is 14 examples. There is 5 examples which gives as result "don't play" and 9 examples which gives as result "will play."

We will thus have $Info(T) = I(9/14, 5/14) = 0.94$

Consider a similar measurement after T has been partitioned in accordance with the n outcomes of a test on the feature X. (in the golfer example, X can be a test on OUTLOOK, TEMPERATURE, ...). The expected information requirement can be found as a weighted sum over the subsets:

$$Info(X,T) = \sum_{i}^{n} \frac{|T_i|}{|T|} Info(T_i) \tag{4}$$

where $T_1, T_2, \ldots, T_m$ is the partition of T induced by the value of X.

In the case of our golfing example, for the attribute Outlook we have:

$$Info(Outlook, T) = 5/14 * I(2/5, 3/5) + 4/14 * I(4/4, 0) + 5/14 * I(3/5, 2/5) = 0.694 \tag{5}$$

Consider the quantity $Gain(X,T)$ defined as:

$$Gain(X,T) = Info(T) - Info(X,T) \tag{6}$$

This represents the difference between the information needed to identify an element of T and the information needed to identify an element of T after the value of attribute X has been obtained, that is, this is the gain in information due to attribute X.

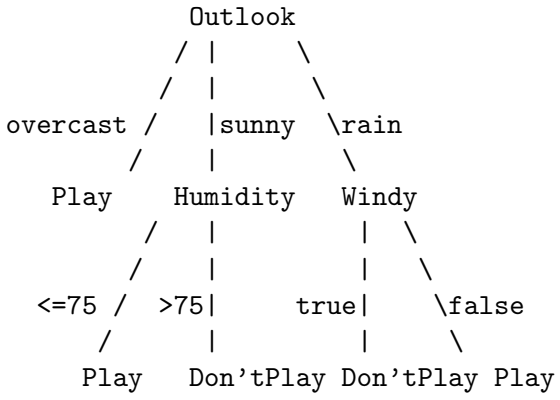
In our golfing example, for the Outlook attribute the gain is:

$$Gain(Outlook, T) = Info(T) - Info(Outlook, T) = 0.94 - 0.694 = 0.246. \tag{7}$$

If we instead consider the attribute Windy, we find that $Gain(Windy, T) = 0.048$. Thus OUTLOOK offers a greater informational gain than WINDY.

We can use this notion of gain to rank attributes and to build decision trees where at each node is located the attribute with greatest gain among the attributes not yet considered in the path from the root.

In the Golfing example we will obtain the following decision tree:



2 Tree Generation: Gain_Ratio criteria.

The notion of Gain introduced earlier tends to favor test on features that have a large number of outcomes (when n is big in equation 4). For example, if we have a feature X that has a distinct value for each record, then $Info(X,T)$ is 0, thus $Gain(X,T)$ is maximal. To compensate for this Quinlan suggests using the following ratio instead of Gain:

$$\text{GainRatio(X,T)} = \frac{\text{Gain(X,T)}}{\text{SplitInfo(X,T)}} \tag{8}$$

.

Consider the information content of a message that indicate not the class to which the case belongs, but the outcome of the test on feature X. By analogy with equation 3, we have

$$SplitInfo(X,T) = -\sum_{i}^{n} \frac{|T_i|}{|T|} \log_2 \frac{|T_i|}{|T|} \tag{9}$$

.

The $GainRatio(X,T)$ is thus the proportion of information generated by the split that is useful for the classification.

In the case of our golfing example $GainRatio(OUTLOOK,T) = 0.246/1.577 = 0.156$ and $GainRatio(WINDY,T) = 0.048/0.985 = 0.049$.

We can use this notion of *GainRatio* to rank attributes and to build decision trees where at each node is located the attribute with greatest *GainRatio* among the attributes not yet considered in the path from the root.

We can also deal with the case of features with continuous ranges. Say that feature X has a continuous range. We examine the values for this features in the training set. Say they are, in increasing order, $A_1, A_2, \ldots, A_m$. Then for each value A_j, $j = 1, 2, ..m$ we partition the records into 2 sets : the first set have the X values up to and including A_j and the second set have the X values greater than A_j. For each of these m partitions we compute the $GainRatio(X(j),T)$ $j = 1, 2, ..m$, and choose the partition that maximizes the gain. If all features are continuous, we will obtain a binary tree.

3 Pruning Trees.

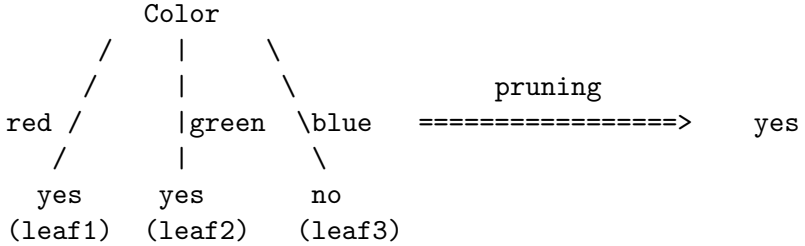
Pruning a tree is the action to replace a whole subtree by a leaf. The replacement takes place if the expected error rate in the subtree is greater than in the single leaf. We will start by generating the whole (generally overfitted) classification tree and simplify it using pruning just after.

The error estimates for leaves and subtrees are computed assuming that they were used to classify a set of unseen cases of the same size as the training set. So a leaf covering N training cases (E of them incorrectly) with a predicted error rate of $U_{CF}(E, N)$ (with U_{CF}: the binomial distribution, CF: the Confidence Level) would give rise to a predicted $N \times U_{CF}(E, N)$ errors. Similarly, the number of predicted errors associated with a (sub)tree is just the sum the the predicted errors of its branches.

An example: Let's consider a dataset of 16 examples describing toys. We want to know if the toy is fun or not.

COLOR	MAX NUMBER OF PLAYERS	FUN
red	2	yes
red	3	yes
green	2	yes
red	2	yes
green	2	yes
green	4	yes
green	2	yes
green	1	yes
red	2	yes
green	2	yes
red	1	yes
blue	2	no
green	2	yes
green	1	yes
red	3	yes
green	1	yes

We obtain the following classification tree:



The predicted error rate for leaf 1 is $6 \times U_{25\%}(0, 6) = 6 \times 0.206$. The predicted error rate for leaf 2 is $9 \times U_{25\%}(0, 9) = 9 \times 0.143$. The predicted error rate for leaf 3 is $1 \times U_{25\%}(0, 1) = 1 \times 0.75$. The total number of predicted error for this subtree is $6 \times 0.206 + 9 \times 0.143 + 1 \times 0.75 = 3.273$. If the tree were replaced by the simple leaf "Yes", the predicted error rate would have been:

$16 \times U_{25\%}(1, 16) = 16 \times 0.157 = 2.512$. Since the simple leaf has a lower predicted error ($2.512 < 3.273$), the tree is pruned to a leaf.

4 Limitations

The major limitation is that the feature space can only be partitioned in boxes parallel to axes of the space.

The implementation of the classification-tree algorithm I realized is only able to classify continuous features. This means, that it can only generate binary trees. We can still use the program with special discrete features like *MAX NUMBER OF PLAYERS=(1,2,3,4)* in the *toy example* above. The discrete feature must exhibit some ordering property. This means that discrete features like *COLOR=(red,green,blue)* are not allowed.

5 Bagging and Feature selection

We can use many classifiers and make a vote to obtain the final class of the examples we want to classify. How does it help? Let's assume you have 10 decision trees $T_1, T_2, \ldots, T_{10}$. You want to make classification in 3 classes C_1, C_2, C_3. Let's now assume that the first 4 trees $T_1, \ldots, T_4$ always recognize and classify correctly class C_1 (they are specialized for C_1). The other trees are specialized in other classes. Let's now assume you want to classify a new example. This example belongs to C_1. The trees $T_1, \ldots, T_4$ will answer C_1. The other trees will gives random answer which will form some "uniformly distributed noise" during the vote. The result of the vote will thus be C_1. We just realized an *error de-correlation*.

How to generate the 10 trees? We don't have to know what are the specialty of each tree. We only have to build trees that have various behavior. We will build the 10 different trees using the algorithm previously described on 10 different sets of examples. How do we generate these 10 sets of example? We will use only a small part (called a bootstrap) of the full set of example (this technique is called BAGGING). Each bootstrap (there are here 10) will be build using random example taken from the full set of example (random selection with duplication allowed). We will also use a small subset of all the features available (this technique is called FEATURE SELECTION). For each bootstrap, we will use different randomly chosen features (random selection with duplication forbidden). The combination of BAGGING and FEATURE SELECTION is called BAGFS.

What's the optimal number of trees? Can we take the highest number possible? no, because another phenomena will appear: the OVERFITING. The classifier lose its generalization ability. It can only classify the examples which were used to build it. It cannot classify correctly unseen examples.